Part I – Strategic Framing & High-Level Architecture

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1 Motivation & Industry Context

Digital commerce is moving toward **high-frequency, sub-cent payments**: AI API calls, IoT data streams, in-game micro-purchases, and more. The current front-runner, **Lightning Network (LN)**, hits three hard walls:

1. **Liquidity opacity** — senders must probe many paths; large payments often fail.
2. **Unpredictable fees / ROI** — routing nodes have no stable return, so capital leaks away.
3. **No inclusion guarantees** — a congested mempool can censor channel-close or dispute TXs.

Hydra tackles these constraints by making **payment responsibility a first-class duty of the validator set** itself, secured by Proof-of-Stake (PoS).

1.1 Strategic Loop – Economics ⇄ Technology

1. **Collateralised validators (PoS + payment duty)** Large slashable stake **plus** a governance-reserved block-space lane ⇒ highly available, well-capitalised nodes.
2. **Deterministic rounds** Because block-space is guaranteed, every round is sized so that *all* tab updates, refreshes, and disputes land on-chain before their deadlines.
3. **Enforceable Hydra topology** With stake on the line we can impose the logarithmic tab suffix-tree routing policy; HTLC auto-closure + slashing steadily remove dishonest peers.
4. **Proof-of-Circulation & predictable ROI** Tokens routed along verifiable paths earn a fixed **τ r** reward each round; a dynamic burn **β(t)** prices congestion transparently.
5. **Token utility & capital fly-wheel** Burns and rewards tie token demand to payment volume, attracting more stake, lowering routing cost, and drawing still more volume.

   This closed loop converts payment routing from a best-effort overlay into a **stake-secured public utility**.

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2 Hydra as a Stand-Alone PoS Chain

2.1 Dual-Purpose Stake

Portion	Share of *s*	Purpose
α · s	20 %	Locked as **liquidity** in deterministic tabs (PoC).
(1 – α) · s	80 %	**Consensus security**; fully slashable.

Static-adversary safety: consensus tolerates up to **(1 – α)/3** stake equivocation; routing capacity degrades by $\leq \alpha f$.

2.2 Reserved Protocol Lane

Governance allocates $\approx$ **1 000 tx s^{-1}** exclusively for:

- Tab refreshes ($\leq k$ per round)
- PoC & balance Merkle-root commits
- Dispute proofs / slashing

Thus **every node can settle all state within one round — guaranteed.**

2.3 Epochic Re-Identification

Nodes must rotate their ID once per epoch (days – weeks) and may reboot at most s times/epoch. This thwarts slow Sybil accumulation while letting honest operators recover keys.

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3 High-Level Architecture

3.1 Deterministic Topology

- Node IDs live in a **Kademlia key-space** (N = 2^ℓ).
- Each node opens **$\ell = \log_2 N$** tabs; each tab holds **1/ℓ** capital and connects to **$q \approx \log N$** peers.

3.2 Two-Layer Routing

1. **Split circulation (C_1)** — each node spreads **1/ℓ** capital uniformly (coin = 1/ℓ N).
2. **Merge circulation (C_2)** — intermediaries forward received coins to final targets.

Any payment matrix obeying per-node send/receive $\leq 1/2\,\ell$ embeds into $C_1 + C_2$ with zero tab overload.

3.3 Proof of Circulation (PoC)

- Public randomness R + token-id $\Rightarrow$ deterministic intermediary z.
- Nodes commit **Merkle roots** of routed tokens each round.
- Auditor samples $\approx$ 150 tokens; wrong paths $\rightarrow$ slashing of stake.

3.4 Economics & Congestion Control

Payment fee formula:

$$\text{Cost} = V\,(\beta + \tau r) + \text{tip}$$

- **β(t)** — EIP-1559-style burn keeps utilisation $\approx$ 50 %.
- **τ r** — opportunity-cost fee for liquidity lock.
- **Inflation** tops up only if total fees \< target ROI R*.

3.5 Lifecycle

Phase	Duration	Key actions
Circulation	1/ℓ of round	Route 1/ℓ budget
Round	ℓ circulations	Commit roots · refresh *k* tabs · settle disputes
Epoch	many rounds	Mandatory ID refresh

3.6 Strategic Edge

L1-exclusive advantages * Stake-backed accountability & predictable ROI
Governance-reserved block-space
Token burns + rewards directly tied to usage

Enhanced (but L2-feasible) capabilities * Deterministic ½ log N capacity per node-pair
Self-healing topology (HTLC pruning + ID rotation)
Lightweight audit (O(log N) proofs)

Hydra delivers **billion-scale liquidity, bounded risk windows, and transparent fees** — without global path search.

Hydra Topology – Ideal Symmetric Case (N = 2^ℓ)

> *This section lays the mathematical foundation for Hydra's routing guarantees in the perfectly symmetric case where the network size is an exact power of two. Subsequent sections generalise to random IDs and churn.*

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1. Preliminaries & Notation

Symbol	Meaning
N	Total nodes, here exactly **2^ℓ**
ℓ	Bit-length of node IDs, $\ell = \log_2 N$
Tab k	The routing table that connects to peers whose IDs share the first **k** bits with the owner and differ in bit *(k + 1)*
q	Number of physical channels stored in each tab (irrelevant in the perfect case; all peers exist)

Each node owns **ℓ tabs**, each holding **1/ℓ** of its capital. Total capital per node is normalised to 1.

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2. Contention-Free Routing Trees

For every node we build a deterministic out-tree that never contends with another node's tree on a given tab.

2.1 Construction

Node *X*'s tree contains one edge per level **k ∈ [0 … ℓ − 1]**:

```
prefix_k(X) · 0···0   →   prefix_{k+1}(X) · 0···0
```

Because each node has a *unique* prefix at every depth, **no two nodes use the same tab edge in the same direction**. ⇒ **contention-free**.

2.2 Greedy Monotone-Suffix Routing

Given a destination *D*, the sender repeatedly flips the next mismatching bit toward *D*. This is:

- **Greedy** — always reduces Hamming distance by 1.
- **Stateless** — needs only the next bit, not global view.
- **Adaptive** — if a hop fails, the sender can pick an alternative peer within the same bucket without recomputing the full path.

3. Uniform One-to-All Traffic

Every node sends **1/(ℓ N)** to **every** other node along the monotone-suffix path.

Lemma 1 (Exact Tab Saturation)

For each node *X* and each tab *k*:

- Self-traffic load = **1/(2^k ℓ)**
- Transit load = **(1 − 1/2^k)/ℓ**
- **Total** = **1/ℓ** (exact tab capacity)

Proof. Count how many destinations share the first *k* bits with *X* (2^{ℓ−k}) and compute share of 1/(ℓ N) each. □

Corollary 1

The uniform one-to-all pattern **fully utilises every tab without overloading any**.

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4. Example (ℓ = 3, N = 8)

IDs: `000, 001, 010, 011, 100, 101, 110, 111`

Node `000` tabs:

Tab	Bucket IDs	Capital	Self load	Transit load
0	100, 101, 110, 111	1/3	1/3·1/2	1/3·1/2
1	010, 011	1/3	1/3·1/4	1/3·3/4
2	001	1/3	1/3·1/8	1/3·7/8

Sums match 1/3 each — confirming the lemma.

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5. Key Properties Recap

1. **Contention-free trees** — each node's routing tree edges are unique; no intra-tab competition.
2. **Greedy monotone routing** — no shortest-path computation; local decision suffices.
3. **Deterministic capacity** — every node can reliably send **1/ℓ** of its capital per circulation, setting the stage for the **double-circulation any-to-any guarantee**.

   Next Section ⇒ **Double Circulation Theorem:** how two such uniform passes allow any node to push up to ½ ℓ^{-1} of capital to any single peer.

Double Circulation Theorem – Ideal Hydra Topology

*This section proves that any valid one-to-one payment plan with bounded per-node spend and receive limits can be embedded using two full Hydra circulations.

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1. Setup and Notation

- Let $N = 2^\ell$ be the number of nodes.
- Each node owns ℓ routing tabs, each holding $1 / \ell$ of its capital.
- A **circulation** is a full one-to-all uniform payment pass of $1 / (\ell N)$ coins to each node.
- Each tab is exactly saturated by this traffic (see prior section).

We now aim to support **any-to-any** payments up to $1 / (2\ell)$ per sender and receiver.

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2. Statement of the Theorem

Theorem (Double Circulation Capacity).\ Any payment plan in which:

- Each sender emits $\leq \frac{1}{2\ell}$ of capital total
- Each receiver accepts $\leq \frac{1}{2\ell}$ of capital total

Can be embedded in two full Hydra circulations (Split + Merge), such that every transfer uses a single intermediary and respects all tab capacities.

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3. Formal Payment-Plan Definition

Let $P = (P_{ij})$ be an $N \times N$ non-negative matrix, where P_{ij} is the amount node i plans to pay node j, and $P_{ii} = 0$.

Define per-node totals:

$$S_i := \sum_j P_{ij} \quad (\text{total sent by node } i) \qquad R_j := \sum_i P_{ij} \quad (\text{total received by node } j)$$

A plan is **valid** when:

$$S_i \leq \tfrac{1}{2\ell}, \quad R_j \leq \tfrac{1}{2\ell} \quad \text{for all } i, j$$

Coin discretisation. Set atomic coin value $c := \frac{1}{2\ell N}$ so each node's send/receive budget is exactly N coins.

4. Existence of an Intermediary — Counting Argument

Consider a coin $(X \to Y)$ that we want to implement via an intermediary Z :

- In **Circulation C_1 (Split)**: $X \to Z$ (adds c to Z 's inbound tally)
- In **Circulation C_2 (Merge)**: $Z \to Y$ (adds c to Z 's outbound tally)

Each node can forward at most N coins total in each circulation.

Proof by Contradiction

Assume no suitable intermediary Z exists for the pair $(X \to Y)$. Then for every node Z , one of the following must hold:

- **Case A — Z already forwarded a coin to Y in C_2:**\ Y has already received $\geq N$ coins → contradicts the receive budget.
- **Case B — X already sent a coin to Z in C_1:**\ X has already sent $\geq N$ coins → contradicts the send budget.

In both cases, the plan would violate its validity conditions. Hence, a valid plan always admits a feasible intermediary.

> *Remark.* This is equivalent to the classical Hall's matching theorem, where a bipartite graph of (X,Y) pairs with capacity constraints always admits a matching when totals align.

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5. Routing Schedule

1. **Circulation C_1 (Split):**\ Every node X sends its assigned coins to intermediaries $Z_1, Z_2, \ldots$.\ Each X sends $\leq \frac{1}{2\ell}$; each Z receives $\leq \frac{1}{2\ell}$.
2. **Circulation C_2 (Merge):**\ Each intermediary Z forwards received coins to targets $Y_1, Y_2, \ldots$.\ Each Z sends $\leq \frac{1}{2\ell}$; each Y receives $\leq \frac{1}{2\ell}$.

Each tab (with capacity $1 \,/\, \ell$) is used only once per circulation ⇒ safe.

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6. Example

- Capital per node = 1
- Per-circulation budget = $1 \,/\, \ell = 1 \,/\, 3$
- Max transfer between any (X, Y) pair = $1 \,/\, (2\ell) = 1 \,/\, 6$

Let X want to send $1/6$ to Y :

- X selects an intermediary Z that has not already received from X and not yet forwarded to Y .
- If no such Z exists, then either X has spent or Y has received $1/3$ total ⇒ contradiction.

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7. Summary

The double-circulation pattern enables any sparse payment plan (bounded at $1/(2\ell)$ per node) to be embedded into a deterministic, contention-free, symmetric network. This provides the foundation for:

- Stable throughput guarantees
- Clean accounting via token traces
- Efficient auditing for incentive mechanisms

Next: Generalising to Randomised Hydra Topology

Randomised Hydra Topology – Throughput and Security Guarantees

The ideal Hydra assumes a perfect power-of-two ID space and full bucket population. Real deployments assign IDs pseudorandomly and each tab stores only ***q ≈ c log N*** *peers, refreshed periodically. This section derives throughput and security guarantees under these realistic assumptions.*

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1 Model Extensions

Symbol	Meaning
N	Number of validators (not necessarily power-of-two)
ℓ = ⌈log₂ N⌉	Logical ID length
bucket(k)	IDs sharing first *k* bits, differing at bit *k + 1*
q	Peers sampled per bucket, $q = c \log N$ ($c \geq 4$)
k_refresh	Refresh attempts per round for a stale tab
m_round	Rounds between refreshes of the same bucket
f	Fraction of nodes under static adversary control

Nodes choose each of their *q* peers **uniformly** from the bucket and may replace up to *k_refresh* peers every round.

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2 Bucket Honesty Concentration

Lemma 1 (Big buckets concentrate)

For k-buckets with expected size $S_k = N/2^{k+1} \gg q$, peer sampling is approximately Binomial(q, f) . Chernoff bound:

$$ \Pr{\text{bad fraction} \ge f+\varepsilon}\;\le\; e^{-2\varepsilon^{2}q}. $$

Choosing **q = Θ(log N)** makes this probability < N^{-2} after a union bound over all ℓN buckets.

Lemma 2 (Small buckets become full mesh)

For depths where $S_k \le 8q$ (high-k buckets), the bucket contains $O(q)$ nodes. With high probability it becomes **fully connected** after one refresh, and adversary share $\le f + \varepsilon$.

Proof idea: sampling without replacement covers the entire small set after $\approx \log S_k$ draws; static adversary cannot skew beyond expectation.

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3 Inbound/Outbound Degree Bounds

Each node's **outbound** degree per bucket is fixed at *q*. The **inbound** degree is Binomial$(Nq, 1/N)$. Chernoff:

$$ \Pr\left[\bigl|\text{in_deg}-q\bigr| > \lambda\right] \le 2e^{-\lambda^{2}/(3q)}. $$

Set $\lambda = \sqrt{q \log N} \rightarrow$ deviation $O(\sqrt{\log N})$ w.h.p. ⇒ per-node usable capacity factor

$$ g = 1-\varepsilon,\quad \varepsilon=O!\bigl(1/\sqrt{\log N}\bigr). $$

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4 Min-Cut and Capacity Guarantee

Karger–Stein random graph bounds imply that with *q = Θ(log N)* and g above, the **edge cut** between any two nodes (or clusters) is at least

$$ C_{\min}\;\ge\; \frac{g\,(1-f-\varepsilon)}{2\log N} $$

with probability ≥ 1 – N^{-c}.

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5 Double Circulation with Loss Factor *g*

Scale every tab budget by **g**. The coin-matching/counting argument from the ideal theorem still works, yielding:

$$ \sum_j P_{ij} \le \frac{g}{2\log N},\quad \sum_i P_{ij} \le \frac{g}{2\log N} \Rightarrow \text{routable in two circulations.} $$

Capacity loss is thus only **ε = O(1/$\sqrt{\log N}$)**.

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6 Refresh Dynamics

Each refresh replaces a peer with fresh sampling. After *k_refresh* replacements:

$$ \Pr[\text{bucket > } (f+\varepsilon)q] \le e^{-2\varepsilon^{2}qk_{\text{refresh}}}. $$

Choose $q\,k_refresh = \Theta(\log^2 N)$ to make failure negligible.

Epochic reboot: every node must change ID once per epoch; adversary cannot accumulate a fully bad bucket before IDs reshuffle.

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7 Practical Parameter Suggestion

Parameter	Rule	Example ($N = 10^5$)
q	$6 \log N$	≈ 85
k_refresh	10	
epoch	$\approx$ 1 week	
Failure prob.	$\le 10^{-4}$ per bucket	
Loss factor *g*	≥ 0.95	

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8 Takeaways

1. **Honesty concentration:** with $q = \Theta(\log N)$, adversary share per bucket stays $\approx f$.
2. **Degree symmetry:** inbound/outbound capacity deviates by only $\sqrt{\log N}$ fraction.
3. **Throughput loss:** double circulation still guarantees $\ge g/(2\log N)$ per node-pair — negligible loss for large N.
4. **Time defence:** refresh + epoch reboot make bucket poisoning exponentially hard.

These results show that Hydra's deterministic capacity guarantees remain robust even with random ID assignment, limited peer lists, and slow churn.

Refresh & Epoch Mechanics in Hydra

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0 Overview

Hydra combines short-term tab refreshes and long-term epoch ID renewals to maintain a clean, honest network structure over time. This component addresses how the system:

- Maintains bucket quality under dynamic churn
- Flushes adversaries through repeated routing failures
- Preserves sybil resistance by bounding ID change rates

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1 Tab Refreshes per Round

Each node maintains ℓ tabs (logarithmic in network size). In every round:

- Node is allowed up to **k refreshes** (e.g., 4–8), each replacing a tab that:
- Routinely fails HTLC forwarding
- Becomes unresponsive

Each tab connects to **q peers**, chosen uniformly from the k-bucket.

> "q" is tuned such that, with high probability, > 1−f of the k-bucket peers are honest even after adversarial selection

Since some tabs connect to large buckets (e.g. k = 0), others to small ones (k = ℓ), the refresh budget k ensures full mesh for small buckets, and statistical convergence to expectations for large ones.

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2 HTLC Failure-Based Flushing

Routing attempts that fail due to corrupted intermediaries trigger one of two responses:

- **Cooperative failures** (honest source detects misbehavior):
- Results in dispute and potential slashing
- Misbehaving tab is refreshed (replaced)
- **Silent failures** (unresponsive peers):
- After retry and timeout, tab is considered faulty and refreshed

With enough circulation rounds, each failure eliminates one bad tab, and for **long-lived honest nodes**, neighborhoods converge to fully honest peers.

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3 Epochs and ID Reboots

To prevent slow adaptive corruption of a bucket:

- Epoch duration = **several hours** (configurable)
- During each epoch:
- Every node must **reboot its ID once** (i.e., get a fresh address)
- But no more than **s total reboots** per node per epoch

This balances:

- Preventing adversaries from fully capturing a bucket by repeated ID churn
- Allowing honest nodes to escape poorly connected regions

Typical values: epoch ~6h, s = 4

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4 Security Guarantees

Let f be the fraction of adversarial stake.

- Each large k-bucket (size ~ N / 2^k) has adversarial concentration ~ f±ε
- After ℓ rounds, all bad tabs flushed from honest node neighborhoods
- Even for small buckets, large q ensures majority honesty with high probability

Adversary cannot concentrate > f of capital in more than a negligible number of tabs simultaneously under ID and refresh constraints.

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5 Parameter Summary

Parameter	Typical Value	Notes
ℓ (tabs/node)	17 (log_2 N)	Tab per k-distance bucket
q (peers/tab)	16–32	Full mesh for small buckets
k (refreshes)	4–8 per round	To flush faulty tabs
s (ID reboots)	3–5 per epoch	Prevent slow-bucket takeover
Epoch length	4–8 hours	Tradeoff: safety vs adaptivity

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6 Conclusion

By combining bounded churn, failure-driven refresh, and ID reboot constraints, Hydra ensures:

- Fast convergence to honest local neighborhoods
- Ongoing resilience against long-term infiltration
- High availability for nodes with stable routing

These dynamics underpin the long-term reliability and security of Hydra’s overlay routing structure.

Auditing & Proof-of-Circulation in Hydra

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0 Synopsis

Hydra enforces payment honesty with a **two-tier probabilistic audit**:

1. **Proof-of-Circulation (PoC)** — rewards nodes for routing tokens along verifiable deterministic paths.
2. **Policy Compliance** — slashes nodes that exceed their per-tab flow caps.

With only $\Theta(\log N)$ random token probes per round, both properties hold with failure probability N^{-c} . This section formalises the mechanism and derives security bounds.

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1 Token Life-Cycle

1. **Mint / deposit** — token of value $c = \frac{1}{\ell N}$ enters a node's liquidity pool.
2. **Path assignment** — deterministic intermediary $z(t) := H(\text{beacon} \,\|\, \text{token_id})$
3. **Forward** — source $A \to$ intermediary $z \to$ destination A (earns PoC reward τr).
4. **Settle** — node commits the token's trace in a Merkle tree at round end.

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2 Commitment Phase (per round)

Each node publishes **one Merkle root** with two disjoint index ranges:

Range	Content	Counter
T_PoC	tokens claimed for reward; full path stored	y_{PoC}
T_Relay	*all* tokens routed (PoC ∪ user)	y_{Relay}

On-chain, only the root and the two counters are stored.

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3 Audit Sampling Protocol

1. Verifier draws m_{PoC} indices uniformly from $[1..y_{\text{PoC}}]$.\ • Checks path = $A \to z(t) \to A$.\ • Verifies signatures & tab sequence.
2. Draws m_{Relay} indices from $[1..y_{\text{Relay}}]$.\ • Computes tab usage of that token; ensures no tab over-budget.
3. Any failure ⇒ node is **slashed**: burns reward and up to $(1-\alpha)s$ stake.

Values m_{PoC} and m_{Relay} are protocol constants (≈ 150).

4 Security Proofs

4.1 PoC Overclaim

If a node exaggerates token count by fraction δ (i.e., lies on δy_{PoC} tokens), detection probability after m_{PoC} probes: $\Pr[\text{miss}] = (1-\delta)^{m_{\mathrm{PoC}}}$ Choosing $m_{\mathrm{PoC}} = 150$ and $\delta = 0.03 \rightarrow \Pr[\text{miss}] < 1\%$.

4.2 Tab Over-Flow

Let a specific tab exceed capacity by $E_k > \varepsilon$ (value). Then at least $p_k = \frac{E_k}{y_{\mathrm{Relay}} c}$ of relay tokens are fraudulent. After m_{Relay} probes, Chernoff gives: $\Pr[\text{no hit}] \le e^{-m_{\mathrm{Relay}} p_k}$ With $m_{\mathrm{Relay}} = 150$ and $p_k \ge 0.03 \rightarrow \le 1\%$.

4.3 Union Over ℓ Tabs

Total evasion probability $\le \ell e^{-m_{\mathrm{Relay}} p_k}$. For $\ell \approx 17$ ($N = 10^5$) this stays $< 10^{-6}$.

5 Economic Consistency

> **Honest routing is a Nash equilibrium.**

Expected reward: $y_{\mathrm{PoC}} \tau r$

Expected penalty if cheating by δ: $\left(1 - (1-\delta)^m\right)(1-\alpha)s$

With $m = 150$ and $(1-\alpha)s \gg y_{\mathrm{PoC}} \tau r$, cheating has negative EV.

Sampling cost: $\Theta(\log N)$ proofs → gas/OPEX sub-linear.

6 Parameter Table & Examples (N = 10^5)

Parameter	Value	Rationale
m_{PoC}	150	>99 % catch for 3 % overclaim
m_{Relay}	150	>99 % catch for 3 % tab overuse
Audit failure prob.	$<10^{-6}$	Union of tabs + token sets
Stake at risk	(1−α)s	Full PoS portion

7 Implementation Notes

- **Single vs dual root:** one tree easier; optional two-root split for parallel verification.
- **Audit initiator:** on-chain interactive protocol.
- **Batch proofs:** over long epoch.

Takeaway

With logarithmic sampling, Hydra **probabilistically enforces** both PoC honesty and per-tab capacity limits, while keeping verification light and slashing heavy — aligning economic incentives with technical guarantees.

Hydra Tokenomics & Incentive Layer

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0 Overview

Hydra nodes earn rewards by routing protocol-assigned tokens along deterministic paths (**Proof-of-Circulation**). Users must pay to bypass this path. Tokenomics are tightly integrated with PoS economics and congestion pricing.

We detail how:

- Nodes split their capital between PoS and routing.
- Burn regulates network congestion.
- Rewards are standardized and fees transparently priced.
- Routing caps ensure predictable ROI for liquidity providers.

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1 Capital Split: PoS vs Routing

Each node stakes a total capital s . This is split into:

- **PoS stake**: $(1-\alpha)s$
- **Routing liquidity**: αs

Only the PoS stake is used to secure consensus and gets slashed upon dishonest behavior. Routing stake is committed to tabs (channels) and earns routing fees.

This split introduces a tradeoff:

- More routing stake → more volume, more reward
- More PoS stake → stronger consensus security

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2 Proof-of-Circulation Reward (protocol reward)

Every round, each node receives up to $y_{extPoC} \cdot r$ reward for tokens that completed a verifiable round-trip.

- r is the **unit reward per token**.
- y_{extPoC} is the number of successfully routed protocol tokens.

Nodes can additionally earn fees from user payments outside PoC.

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3 Transparent Standardized Pricing & Congestion Control

Hydra uses **EIP-1559-style fee adjustment**:

- Each protocol token routed incurs a **burn** b .
- The **base reward** r is adjusted over time to maintain steady congestion.

Because protocol tokens follow deterministic paths, the **opportunity cost** of using them is well-defined:

- Routing one token denies earning r elsewhere.
- Locking one token for au time denies protocol participation → cost = aur

Thus:

- **User fees** must exceed this opportunity cost to incentivize routing.
- **Burn rate** determines reward level r , which regulates congestion.

This pricing is transparent and standardized — it is globally visible and predictable.

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4 Auditing & Honest Equilibrium

Hydra nodes commit a Merkle tree of all routed tokens. A logarithmic number of random probes verify:

- PoC honesty (correct round-trip path)
- Tab policy compliance (no overuse)

Slashing risk on PoS stake makes cheating economically irrational.

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5 Liquidity Incentives & Predictability

Because:

- Routing capacity per node is capped
- Paths are deterministic
- Rewards are proportional to usage

→ The ROI is **stable and tied to network usage**. This allows:

- Nodes to benchmark expected returns
- Capital to enter until marginal return = opportunity cost

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6 Token Value Flywheel

1. **Protocol usage** drives token burns.
2. **Burn** drives higher per-token reward r .
3. **Reward** attracts routing capital, and manages congestion
4. **Capital** increases throughput and reliability.

This positive loop gives the token monetary value tied directly to utility.

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7 Parameters & Levers

Symbol	Meaning	Adjusted by
α	PoS vs routing split	Governance
r	Reward per protocol token	Auto (burn-based)
b	Burn per routed token	Governance
au	Token lock duration	Protocol config

These allow tuning reward stability, security, and congestion adaptively.

Takeaway

Hydra's incentive layer leverages PoS security, transparent pricing, and deterministic token routing to create a stable, predictable, and congestion-aware reward system — aligning liquidity providers with protocol goals.

Hydra: Optimizations & Future Research

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1 Batchable Channel Updates

Hydra minimizes on-chain bandwidth by batching multiple tab and user channel updates into a single transaction per node per round. This includes:

- A serialized list of balances for all active tabs (channels with peers).
- User and merchant balance commitments, updated once per round.
- Refresh instructions for each tab.

This design offers:

- **Efficiency**: reduces on-chain transactions from $O(l \cdot q)$ to $O(1)$ per node per round.
- **Accountability**: since claims are disputeable, any misreporting risks collateral.

Any peer or client who detects a misreported value can initiate an on-chain challenge, referencing the original Merkle root commitment and providing proof of mismatch.

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2 Future Directions

Hydra opens several promising research avenues:

2.1 Dynamic Reward Tuning

Explore mechanisms beyond EIP-1559 for setting the burn-adjusted reward per token routed. For instance: - Multi-objective pricing based on both congestion and fairness. - Stochastic modeling of burn–reward equilibrium.

2.2 User Payment Path Compression

Design compression schemes for user-initiated multi-hop payments to: - Reduce payment metadata overhead. - Improve privacy without breaking auditing guarantees.

2.3 Dispute Compression

- Aggregate multiple channel disputes into a single proof object.
- Compress witness data via SNARKs or similar zero-knowledge methods.

2.4 Adaptive Tab Structures

Allow node tab counts to vary with: - Routing volume. - Node performance. - Peer churn rate.

This may improve throughput while preserving uniformity guarantees.

2.5 Stake Compounding and Epoch Extensions

- Reward nodes with bonus returns for locking capital across many epochs.
- Introduce long-term staking pools for delegators to fund payment routers.

2.6 Fairness and Graph Bias Audits

- Detect tab refresh biases using statistical fairness tests.
- Audit topology skew caused by adversarial clustering.

2.7 Adversary Modeling Over Epochs

- Formalize resistance to partially adaptive adversaries.
- Model time-based bucket corruption and recovery.

Summary

Hydra's foundation enables powerful optimizations and novel research. From micro-level compression to macro-level economic design, future improvements can extend its efficiency, scalability, and robustness across diverse payment use cases.

Hydra: Throughput & Security Metrics (High-Capital L1 Setup)

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1 Representative Parameter Set

Symbol	Meaning	Value
N	Total number of nodes	50,000
q	Peers per bucket (tab)	85
ℓ	Number of buckets per node ($\log N$)	\~15.6
k	Refresh attempts per round per tab	3
α	Routing capital ratio	0.2
s	Stake per node	\$20,000
τ_R	Round duration	5 minutes
T_{epoch}	Epoch duration	1–2 days
b	Burn per routed token	variable
r	Reward per routed protocol token	Adjusted by burn

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2 Throughput Estimates

2.1 Node-Level Throughput

Each node has $\alpha s = \$4{,}000$ in routing capital.

- Per circulation: can send and receive $\frac{1}{\ell} = \frac{1}{15.6}$ of capital = \~\$256
- Per round: $\ell = 15.6$ circulations → \$4,000
- Per second: \$4,000 / 300 seconds ≈ **\$13.33/s per node**

2.2 Network-Level Throughput

- Total network: \$13.33/s × 50,000 = **\$666,500/s**
- Daily throughput: \$57.6 billion/day

This volume is **fully auditable and congestion-aware**.

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3 Security Metrics

3.1 Bucket-Level Adversary Penetration

Assume a static adversary controls 33% of nodes.

- For high-k buckets (size $\approx \log N$), full-mesh with $q = 85$ gives high redundancy.
- With $k = 3$ refresh attempts per tab:
- Probability of missing all honest peers $\approx (1 - 0.67)^3 = 0.037$
- Probability of honest connectivity > 96.3%

3.2 Audit Soundness

Audits use random sampling over Merkle-committed routed tokens.

- Suppose node claims $y = 1000$ tokens routed
- Auditor samples $t = 20$ at random
- If node cheats on ≥ 10 % of tokens:
- Probability of catching ≥1 bad token > **87%**
- With 50 samples → detection > **99.5%**

3.3 Effective Slashable Stake

- Honest node: PoS stake = $(1 - \alpha) \cdot s = \$16{,}000$
- Attacker: controls 33% of 50k nodes → \$264M slashable capital

Even with partial audits, the cost of misreporting becomes prohibitive.

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4 Takeaway

Hydra, with 50k PoS-style nodes at \$20k each, offers:

- \$57B/day in sustainable, verifiable payment throughput
- 99% attack detection probability with light auditing
- Strong incentives for honest participation via burn-reward coupling
- Scalable with low-latency (5-minute) deterministic rounds